Arezo GHAHGHAEI, PhD Candidate
E-mail: arezoghahghaei@gmail.com
Department of Industrial Engineering
Alzahra University, Tehran, Iran
Associate Professor Mehdi SEIFBARGHY, PhD
E-mail: m.seifbarghy@alzahra.ac.ir
Alzahra University, Tehran, Iran

APPROXIMATE ANALYSIS AND SIMULATION OF A THREE-ECHELON INVENTORY SYSTEM WITH ORDER SPLITTING BETWEEN TWO SUPPLIERS

Abstract. *In this paper we consider a three-echelon supply chain with two suppliers, a central warehouse and an arbitrary number of retailers with continuous review policy. Retailers face independent Poisson demand and lead times are stochastic with no predetermined probability distribution. Unsatisfied demand is lost at the retailers and backlogged at the warehouse and suppliers. We restrict the reorder point to be greater than or equal to -1. Orders placed from the warehouse are divided between the two suppliers. In this paper, multi-echelon inventory control and order splitting problems are considered as an integrated model. Adding suppliers as the third echelon to the inventory system causes order crossover at the warehouse. The total cost of the three-echelon inventory system is expressed as a weighted mean of costs for one-for-one policies. To assess the accuracy of the model, the total cost of the mathematical model is compared with that of the simulation.*

Keywords*: Multi-echelon Inventory System, Continuous Review, Lost sales, Order splitting, Simulation.*

JEL Classification: M11, C61, C63

1. Introduction and Literature Review

In today's competitive environment, companies are facing numerous challenges to decrease operational costs, increase profits and maintain their competitive capabilities. Managers have to seize every opportunity to improve their business processes and to improve the performance of the entire supply chain. Inventory control is one of the attractive subjects of research in the supply chain management; inventory control focuses on issuing right order quantity at the right time to reduce the system cost and improve the responsiveness of the supply chain. Due to the increased competition and high cost of unsatisfied demands, multi-echelon

DOI: 10.24818/18423264/54.4.20.15

inventory models have become a favorite topic for analysis (Hajiaghaei-Keshteli and Sajadifar, 2010). Many recent studies have focused on the cost function and the optimal ordering policy in multi-echelon inventory systems under various conditions. Kok et al. (2018) and Ma et al. (2019) presented comprehensive literature review on stochastic inventory models.

In addition to inventory control, sourcing decisions are the main challenge of supply chain management. Since these challenges are mutually dependent, considering them in an integrated model can influence greatly on operating costs and supply chain efficiency; therefore, many firms tend to incorporate their supply decisions into ordering policy to reduce their cost and improve the quality and the service level. In the following, we briefly call the integrated model of inventory control and order splitting problems as the integrated model. Although considerable researches have been devoted to the integrated model, less attention has been paid to multi-echelon models in case of stochastic lead time (Chang and Chang (2017), Bagul and Muhkerjee (2019), Cao and Yao (2019), Duan and Ventura (2019) and Knour et al. (2016)). These researches investigated the integrated model while no shortages are allowed. Other researchers extended previous studies to stochastic lead time while unsatisfied demands are backordered at the retailers (Sculli and Wu (1981), Abginechi et al (2013) and Song et al. (2014)). Hill (1996) and Fong et al. (2000) studied the integrated model for the case of lost sales. Recent articles investigated the integrated model assuming that orders arrive in the same sequence as they were ordered and did not consider the order crossover. However, multi sourcing in the condition of stochastic lead time may cause order crossover (Reizebos, 2006). This paper considers the ordering policy and supply decisions in a three-echelon inventory system including two suppliers, a central warehouse and arbitrary number of identical retailers. Transportation times between all facilities are assumed to be constant while random delay occurs due to the stock out at the warehouse and suppliers. Adding suppliers as third echelon to the supply chain while the lead times are with randomness (because of the addressed delay) results in order crossover at the warehouse. Each sequence of order arrival at warehouse affects the inventory level as well as the operating cost.

The remainder of the paper is organized as follows: Notations and problem formulation are given in Section 2 for the three-echelon supply chain including two suppliers, a central warehouse and arbitrary number of identical and independent retailers with continuous review ordering policy. The optimal results of the mathematical model are presented and examined by simulation in Section 3. Finally, conclusion and future research opportunities are given in Section 4.

2. Problem definition

In this paper, noting the literature and the given gap in the literature, a three-echelon supply chain including two suppliers, a central warehouse and an arbitrary number of identical retailers is considered. Transportation times between all facilities are assumed to be constant while random delay on order shipment may

DOI: 10.24818/18423264/54.4.20.15

occur due to the stock out at the suppliers and the warehouse. The retailers observe a Poisson order arrival process and unsatisfied demand at each retailer is assumed to be lost. Retailers place orders at the central warehouse based on the continuous review inventory policy; the central warehouse and both suppliers follow the same inventory control policy. Suppliers are assumed to have limited capacity in order to fulfill orders received from the warehouse. Orders received from the retailers at the warehouse or from the warehouse at the suppliers are replenished either immediately or after a random delay if backordered; delayed orders are satisfied based on first-in first-out policy. Each supplier places order from an outside source with ample capacity. The independent and dependent decision variables of the model are presented in Table 1; model parameters are given in Table 2.

Table 1. Decision variables of the model

variable	Description
Q_{s1}	Order quantity at supplier 1
R_{s1}	Reorder point at supplier 1 ($R_{s1} \geq -1$)
Q_{s2}	Order quantity at supplier 2
R_{s2}	Reorder point at supplier 2 ($R_{s2} \geq -1$)
Q_{w1}	Part of the warehouse order quantity placed at supplier 1
Q_{w2}	Part of the warehouse order quantity placed at supplier 2
Q_w	Order quantity at the warehouse
R_w	Reorder point at the warehouse ($R_w \geq -1$)
Q_r	Order quantity at each retailer *r*
R_r	Reorder point at each retailer *r*
λ_w	Demand rate at the warehouse
λ_{s1}	Demand rate at supplier 1, $\lambda_{s1} = (Q_{w1}/Q_w)\lambda_w$
λ_{s2}	Demand rate at supplier 2, $\lambda_{s2} = (Q_{w1}/Q_w)\lambda_w$

Table 2. Parameters of the model

Parameters	Description
N	Number of retailers
λ_r	Demand rate at each retailer *r*
L_{s1}	Lead time of supplier *1* orders
L_{s2}	Lead time of supplier *2* orders ($L_{s1} \leq L_{s2}$)
L_{w1}	Transportation time from supplier *1* to the warehouse
T_{s1}	The delay of received orders at supplier *1*
L_{w2}	Transportation time from supplier *2* to the warehouse ($L_{w1} \leq L_{w2}$)
T_{s2}	The delay of received orders at supplier *2*
T_w	Delay of received orders at the warehouse

DOI: 10.24818/18423264/54.4.20.15

L_r	Transportation time from warehouse to each retailer *r*
TC	Expected total cost of the inventory system
K_1^r	Each retailer *r*'s cost per unit where supplier *1* shipment is delivered first
K_2^r	Each retailer *r*'s cost per unit where supplier *2* shipment is delivered first
K_1^{ws}	The warehouse and suppliers' cost per unit where supplier *1* shipment is delivered first
K_2^{ws}	The warehouse and suppliers' cost per unit where supplier *2* shipment is delivered first
h_r	The holding cost per unit and time unit at each retailer *r*
h_w	The holding cost per unit and time unit at the warehouse
h_{s1}	The holding cost per unit and time unit at the supplier *1*
h_{s2}	The holding cost per unit and time unit at the supplier *2*
β_r	Penalty cost per unit lost at each retailer *r*
β_w	Penalty cost per unit lost at the warehouse
Cap_1	Maximum capacity of supplier *1* for each shipment to the warehouse
Cap_2	Maximum capacity of supplier *2* for each shipment to the warehouse

Note that when $R_{s\eta}$ $(\eta = 1{,}2)$ and R_w are less than -1, the problem should be handled in a different way; therefore, we restrict the reorder point to be greater than or equal to -1.

In this research, the total operational cost of the three-echelon supply chain operating under continuous review inventory policy is obtained based on the weighted mean of costs for one-for-one policies. Unit cost function in a serial system with base stock policy is used to derive the weighted mean of costs for one-for-one policies; therefore, we initially give the unit cost function in a three-echelon supply chain which operates under the base-stock policy.

2.1. The unit cost of the serial inventory system operating based on the base stock policy

In this subsection, we consider a basic system including single retailer, a central warehouse and single supplier operating under base stock policy. The inventory holding and shortage costs per unit are obtained for the given basic supply chain. In subsection 3.2, this cost will be used for obtaining the total cost of a three-echelon supply chain including a number of identical retailers, a central warehouse and two suppliers. In the given basic supply chain, the total cost per unit at the retailer, the central warehouse and the supplier is obtained from Eq. (1) in which S_r, S_w and S_s represent the inventory positions at the retailer, the warehouse and the supplier, respectively:

$$c(S_r, S_w, S_s) = \Pi_{ws}(S_r, S_w, S_s) + \Pi_r(S_r, S_w, S_s) \quad (1)$$

DOI: 10.24818/18423264/54.4.20.15

The given cost function in Eq. (1) involves the warehouse and supplier's cost per unit (i.e. $\Pi_{ws}(S_r, S_w, S_s)$) and the retailer's cost per unit (i.e. $\Pi_r(S_r, S_w, S_s)$). $\Pi_{ws}(S_r, S_w, S_s)$ is composed of two components as given in Eq. (2).

$$\Pi_{ws}(S_r, S_w, S_s) = \Pi_w(S_w, S_s) + \Pi_s(S_s) \tag{2}$$

The first component (i.e.$\Pi_w(S_w, S_s)$) represents the warehouse cost per unit and $\Pi_s(S_s)$ represents the supplier's cost per unit. Unsatisfied demands at the warehouse and supplier are assumed to be backlogged. Since the system operates under the base stock policy and each order is placed due to a corresponding demand, if the ordered unit arrives before its corresponding demand, it is assumed to be held in stock and the system incurs holding cost and if it arrives after its corresponding demand, the demand unit is satisfied with a delay of t and incurs shortage cost (Hajiaghaei-Keshteli and Sajadifar, 2010). The delay density function ($f(t)$) in each facility applying base-stock policy can be given as in Eq. (3) in which λ represents the rate of Poisson demand and S represents the inventory position (Axsater, 1990).

$$f(t) = \frac{\lambda^S (L-t)^{S-1} e^{-\lambda(L-t)}}{(S-1)!} \tag{3}$$

For $t = 0$, the probability distribution function is given by Eq. (4).

$$P(t = 0) = \sum_{k=1}^{S} \frac{(\lambda t)^k}{k!} e^{-\lambda t} \tag{4}$$

At the retailer side, if the ordered unit is to be received before its corresponding demand, it is held in stock and a holding cost is added to the system cost. Unsatisfied demand at the retailer is considered to be lost. When the demand is not satisfied immediately, the retailer receives the ordered unit with a delay which incurs the lost demand cost. Retailer's cost in Eq. (1) can be written as in Eq. (5).

$$\Pi_r(S_r, S_w, S_s) = TH_r(S_r, S_w, S_s)(1 - P(S_r)) + \beta_r\, P(S_r) \tag{5}$$

Since unsatisfied demand at the retailer is assumed to be lost, the queuing system under Poisson demand follows M/G/S/S queuing model with S servers and generally distributed service time. In the above Equation, $P(S_r)$ represents the Erlang`s loss formula while S_r servers are occupied. In this system, the arrival rate is equal to λ_r and the mean service time is $\bar{L}_r$, which is the mean lead time at retailer *r* (transportation time plus mean delay due to the warehouse stock out). The first statement in Eq. (5) (i.e. $TH_r(S_r, S_w, S_s)$) represents the expected holding cost per unit at retailer *r*. In the second part, β_r represents the penalty cost per lost demand at retailer *r* and $P(S_r)$ is the probability of lost demand occurrence.

DOI: 10.24818/18423264/54.4.20.15

2.2. Extension to three-echelon inventory system operating under continuous review policy

In this subsection, the mathematical formulations are given to estimate the cost of an inventory system including a number of identical retailers, a central warehouse and two suppliers operating under the continuous review policy. The system cost is determined using the cost of the basic supply chain given in subsection 2.1. In this system, unsatisfied demands at the warehouse and suppliers are assumed to be backordered and random delays incurs due to shortage of stock at these facilities. If the warehouse is out of stock, the demand at the warehouse is satisfied with a delay of T_w. If the suppliers are out of stock, the demand at supplier *1* is satisfied with a delay of T_{s1} and the demand at supplier *2* is met with a delay of T_{s2}. If the demand is satisfied immediately at the suppliers, the delay in order replenishment is equal to zero; otherwise, the delay will be higher than zero. We consider two suppliers in the proposed model and according to the delay value at each supplier, the following four cases may occur:

1. Case 1 :$T_{s1} = t_1$ and $T_{s2} = t_2$
2. Case 2 :$T_{s1} = t_1$ and $T_{s2} = 0$
3. Case 3 :$T_{s1} = 0$ and $T_{s2} = t_2$
4. Case 4 :$T_{s1} = 0$ and $T_{s2} = 0$

Due to the stochastic nature of the delay which occurs at the suppliers, orders will be delivered to the warehouse in different sequences in each ordering cycle. Each delivery sequence affects the inventory level and consequently the operating cost at the warehouse. Accordingly, for each of the above mentioned cases, we face at least one of the two following situations at the warehouse:

Situation *1*: Order from supplier *1* arrives earlier than that of *2* at the warehouse (Fig. 1).

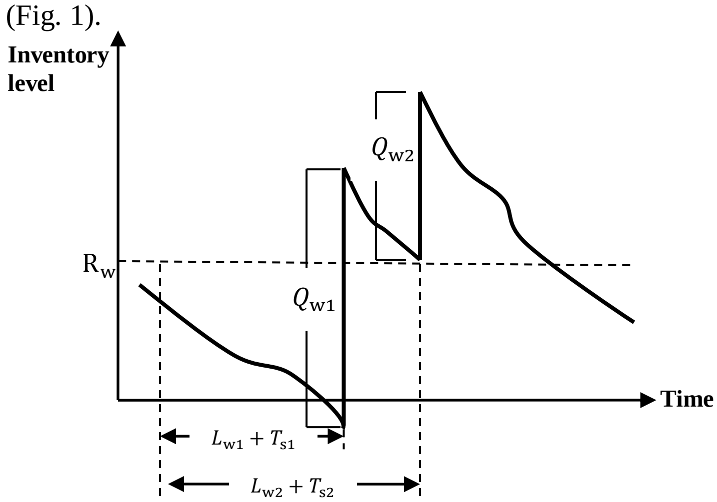

Figure 1. The Warehouse inventory level (Situation 1)

DOI: 10.24818/18423264/54.4.20.15

Situation *2*: Order from supplier *2* arrives earlier than that of *1* at the warehouse (Fig. 2).

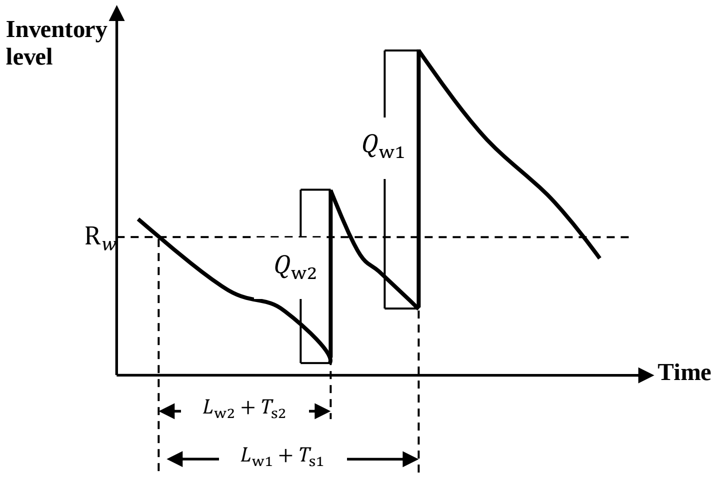

Figure 2. The Warehouse inventory level (Situation 2)

The above four mentioned cases are detailed as follows:

2.2.1. Case 1: $T_{s1} = t_1 > 0$ and $T_{s2} = t_2 > 0$

Considering P_1 as the probability of first case occurrence, $P_1 = P(T_{s1} = t_{s1} > 0) \times P(T_{s2} = t_{s2} > 0)$. In case 1, we may face both Situation *1* and Situation *2*.

2.2.1.1. Situation 1: Order from supplier 1 arrives earlier at the warehouse compared with supplier 2

If the first arriving order comes from supplier *1*, the average cost per unit at the retailer r is equal to K_1^r and the average cost per unit at the warehouse and suppliers is K_1^{ws} while this situation occurs with a probability of $S1 = P(L_{w1} + t_{s1} < L_{w2} + t_{s2})$. Each of the mentioned probabilities can be evaluated using delay density function at the supplier *1* ($f(t_{s1})$) and supplier *2* ($f(t_{s2})$) given in Eq. (3). In Situation *1*, the average cost per unit at the retailer r (K_1^r) can be given as in Eq. (6) (Appendix. A):

$$K_1^r = \frac{1}{Q_{s1}Q_wQ_r}\sum_{k=R_{s1}+1}^{R_{s1}+Q_{s1}}\sum_{j=R_w+1}^{R_w+Q_{w1}}\sum_{i=R_r+1}^{R_r+Q_r} c_{ijk}^1 + \frac{1}{Q_{s2}Q_wQ_r}\sum_{k=R_{s2}+1}^{R_{s2}+Q_{s2}}\sum_{j=R_w+Q_{w1}+1}^{R_w+Q_{w1}+Q_{w2}}\sum_{i=R_r+1}^{R_r+Q_r} c_{ijk}^2 \tag{6}$$

In Eq. (6), the unit cost at the retailer c_{ijk}^{η} ($\eta = 1{,}2$) is obtained from Eq. (7):

DOI: 10.24818/18423264/54.4.20.15

$$c_{ijk}^{\eta} = \sum_{l=0}^{\infty} \Pi_r(i,l,kQ_{w\eta}Q_r) \times p_{lj} \qquad ,\eta = 1,2 \tag{7}$$

And the average cost per unit at the warehouse and suppliers (K_1^{ws}) can be given as in Eq. (8):

$$K_1^{ws} = \frac{1}{Q_{s1}Q_wQ_r} \sum_{k=R_{s1}+1}^{R_{s1}+Q_{s1}} \sum_{j=R_w+1}^{R_w+Q_{w1}} \sum_{i=R_r+1}^{R_r+Q_r} d_{ijk}^1 + \frac{1}{Q_{s2}Q_wQ_r} \sum_{k=R_{s2}+1}^{R_{s2}+Q_{s2}} \sum_{j=R_w+Q_{w1}+1}^{R_w+Q_{w1}+Q_{w2}} \sum_{i=R_r+1}^{R_r+Q_r} d_{ijk}^2 \tag{8}$$

In Eq. (8), the unit cost at the warehouse and the supplier η, d_{ijk}^{η} ($\eta = 1,2$) is calculated as in Eq. (9):

$$d_{ijk}^{\eta} = \sum_{l=0}^{\infty} \Pi_{ws}(i,l,kQ_{w\eta}Q_r) \times p_{lj} \qquad ,\eta = 1,2 \tag{9}$$

$\Pi_r(i,l,kQ_{w\eta}Q_r)$ in Eq. (8) represents the retailer's cost per unit and $\Pi_{ws}(i,l,kQ_{w\eta}Q_r)$ in Eq. (9) is the warehouse and supplier's cost per unit as mentioned in subsection 2.1; while p_{lj} can be calculated as Eq. (10) (Axsater, 1993):

$$p_{lj} = \binom{l-1}{j-1}\left(\frac{Q_r-1}{Q_r}\right)^{l-j}\left(\frac{1}{Q_r}\right)^{j} \tag{10}$$

2.2.1.2. Situation 2: Order from supplier 2 arrives earlier at the warehouse compared with supplier 1

If the first order comes from supplier *2*, the average cost per unit at the retailer r is equal to K_2^r and the average cost per unit at the warehouse and suppliers is K_2^{ws} while this situation occurs with a probability of $S2 = P(L_{w1} + t_{s1} > L_{w2} + t_{s2})$. In this situation, the average cost per unit at the retailer r (K_2^r) can be given as in Eq. (11)

$$K_2^r = \frac{1}{Q_{s2}Q_wQ_r} \sum_{k=R_{s2}+1}^{R_{s2}+Q_{s2}} \sum_{j=R_w+1}^{R_w+Q_{w2}} \sum_{i=R_r+1}^{R_r+Q_r} c_{ijk}^2 + \frac{1}{Q_{s1}Q_wQ_r} \sum_{k=R_{s1}+1}^{R_{s1}+Q_{s1}} \sum_{j=R_w+Q_{w2}+1}^{R_w+Q_{w1}+Q_{w2}} \sum_{i=R_r+1}^{R_r+Q_r} c_{ijk}^1 \tag{11}$$

The average cost per unit at the warehouse and suppliers (K_2^{ws}) can be given as in Eq. (12):

DOI: 10.24818/18423264/54.4.20.15

$$K_2^{ws} = \frac{1}{Q_{s2}Q_wQ_r}\sum_{k=R_{s2}+1}^{R_{s2}+Q_{s2}}\sum_{j=R_w+1}^{R_w+Q_{w2}}\sum_{i=R_r+1}^{R_r+Q_r} d_{ijk}^2 + \frac{1}{Q_{s1}Q_wQ_r}\sum_{k=R_{s1}+1}^{R_{s1}+Q_{s1}}\sum_{j=R_w+Q_{w2}+1}^{R_w+Q_{w1}+Q_{w2}}\sum_{i=R_r+1}^{R_r+Q_r} d_{ijk}^1 \tag{12}$$

In Eqs. (11)-(12), c_{ijk}^{η}, $\eta = 1{,}2$ is calculated as in Eq. (7) and d_{ijk}^{η}, $\eta = 1{,}2$ is obtained from Eq. (9). As unsatisfied demand at each retailer is lost, the demand rates at the warehouse and both suppliers are less than the demand rate at the retailers. Therefore, the retailers' cost per time unit and the warehouse and suppliers' cost per time unit are calculated separately. Finally, the system cost per time unit in Case *1* is given by Eq. (13):

$$TC_1 = N\lambda_r \times (K_1^r \times S1 + K_2^r \times S2) + \lambda_w Q_r \times (K_1^{ws} \times S1 + K_2^{ws} \times S2) \tag{13}$$

2.2.2. Case 2: $\boldsymbol{T_{s1} = t_1} > 0$ and $\boldsymbol{T_{s2} = 0}$

In this case, similar to case *1*, both Situation *1* and Situation *2* may occur. Considering P_2 as the probability of case *2* occurrence, $P_2 = P(T_{s1} = t_{s1} > 0) \times P(T_{s2} = 0)$. $P(T_{s1} = t_{s1} > 0)$ can be calculated based on the delay density function at the supplier *1* ($f(t_{s1})$) and supplier *2* ($f(t_{s2})$) as given in Eq. (3) .Using Eq. (4), $P(T_{s2} = 0)$ can be obtained as in Eq. (14):

$$P(T_{s2} = 0) = \sum_{k=0}^{S_{s2}-1} \frac{\lambda_{s2}^k L_{s2}^k}{k!} e^{\lambda_{s2}L_{s2}} \tag{14}$$

Where S_{s2} is the inventory position at the supplier *2*.

2.2.2.1. Situation 1: Order from supplier 1 arrives earlier at the warehouse compared with supplier 2

If the supplier *1*'s shipment arrives earlier, the average cost per unit at the retailer r (K_1^r) is calculated using Eq. (6) and the average cost per unit at the warehouse and suppliers (K_1^{ws}) is obtained from Eq. (8).

2.2.2.2. Situation 2: Order from supplier 2 arrives earlier at the warehouse compared with supplier 1

If the supplier *2*'s shipment arrives earlier, the average cost per unit at the retailer r (K_2^r) is calculated using Eq. (11) and the average cost per unit at the warehouse and suppliers (K_2^{ws}) can be obtained from Eq. (12). The system cost per time unit in the case *2* is given by Eq. (15):

$$TC_2 = N\lambda_r \times (K_1^r \times S1 + K_2^r \times S2) + \lambda_w Q_r \times (K_1^{ws} \times S1 + K_2^{ws} \times S2) \tag{15}$$

2.2.3. Case 3: $\boldsymbol{T_{s1}} = 0$ and $\boldsymbol{T_{s2} = t_2} > 0$

In this case, the delay at the supplier *1* is zero and $L_{w1} \le L_{w2}$; therefore, the first arriving order comes from supplier *1* and Situation *1* occurs with probability of $S1 = 1$. Denote P_3 as the probability of case *3* occurrence, then, $P_3 = P(T_{s1} = 0) \times P(T_{s2} = t_{s2} > 0)$. $P(T_{s2} = t_{s2} > 0)$ is evaluated using the

DOI: 10.24818/18423264/54.4.20.15

delay density function at supplier *1* (i.e. $f(t_{s1})$) and supplier *2* (i.e. $f(t_{s2})$) given in Eq. (3). Using Eq. (4), $P(T_{s1} = 0)$ can be calculated using Eq. (16):

$$P(T_{s1} = 0) = \sum_{k=0}^{S_{s1}-1} \frac{\lambda_{s1}^k L_{s1}^k}{k!} e^{\lambda_{s1} L_{s1}} \tag{16}$$

Where S_{s1} represents the inventory position at the supplier *1*. Since the first arriving order comes from supplier *1*, then the expected cost per unit at the retailer r (K^r) can be given by Eq. (17):

$$K^r = \frac{1}{Q_{s1} Q_w Q_r} \sum_{k=R_{s1}+1}^{R_{s1}+Q_{s1}} \sum_{j=R_w+1}^{R_w+Q_{w1}} \sum_{i=R_r+1}^{R_r+Q_r} c_{ijk}^1 + \frac{1}{Q_{s2} Q_w Q_r} \sum_{k=R_{s2}+1}^{R_{s2}+Q_{s2}} \sum_{j=R_w+Q_{w1}+1}^{R_w+Q_{w1}+Q_{w2}} \sum_{i=R_r+1}^{R_r+Q_r} c_{ijk}^2 \tag{17}$$

The expected cost per unit at the warehouse and suppliers (K^{ws}) can be given as in Eq. (18):

$$K^{ws} = \frac{1}{Q_{s1} Q_w Q_r} \sum_{k=R_{s1}+1}^{R_{s1}+Q_{s1}} \sum_{j=R_w+1}^{R_w+Q_{w1}} \sum_{i=R_r+1}^{R_r+Q_r} d_{ijk}^1 + \frac{1}{Q_{s2} Q_w Q_r} \sum_{k=R_{s2}+1}^{R_{s2}+Q_{s2}} \sum_{j=R_w+Q_{w1}+1}^{R_w+Q_{w1}+Q_{w2}} \sum_{i=R_r+1}^{R_r+Q_r} d_{ijk}^2 \tag{18}$$

In Eq. (17), c_{ijk}^1 and c_{ijk}^2 are calculated as in Eq. (7) and in Eq. (18), d_{ijk}^1 and d_{ijk}^2 are calculated as in Eq. (9). Finally, the system cost per time unit in case *3* is obtained from Eq. (19):

$$TC_3 = N\lambda_r K^r + \lambda_w Q_r K^{ws} \tag{19}$$

2.2.4. Case 4: $T_{s1} = 0$ and $T_{s2} = 0$

In this case, the delays at both the suppliers are equal to zero and $L_{w1} \leq L_{w2}$; therefore, the first arriving order comes from supplier *1* and Situation *1* occurs with the probability of $S1 = 1$. Defining P_4 as the probability of case *4* occurrence, $P_4 = P(T_{s1} = 0) \times P(T_{s2} = 0)$. Considering Eq. (4), $P(T_{s1} = 0)$ can be given as in Eq. (20):

$$P(T_{s1} = 0) = \sum_{k=0}^{S_{s1}-1} \frac{\lambda_{s1}^k L_{s1}^k}{k!} e^{\lambda_{s1} L_{s1}} \tag{20}$$

Where S_{s1} represents the inventory position at the supplier *1* and $P(T_{s2} = 0)$ is obtained from Eq. (21):

$$P(T_{s2} = 0) = \sum_{k=0}^{S_{s2}-1} \frac{\lambda_{s2}^k L_{s2}^k}{k!} e^{\lambda_{s2} L_{s2}} \tag{21}$$

DOI: 10.24818/18423264/54.4.20.15

Where S_{s2} represents the inventory position at supplier *2*. In this case, the expected cost per unit at the retailer r (K^r) is calculated using Eq. (17) and the expected cost per unit at the warehouse and suppliers (K^{ws}) is obtained from Eq. (18). Therefore, the system cost per time unit for case *4* can be obtained from Eq. (22):

$$TC_4 = N\lambda_r K^r + \lambda_w Q_r K^{ws} \tag{22}$$

Finally, the expected cost of the given inventory system and the related constraints considering the above-mentioned cases can be given as in Eqs. (23)-(29):

$$TC = TC_1 \times P_1 + TC_2 \times P_2 + TC_3 \times P_3 + TC_4 \times P_4 \tag{23}$$

$s.t$:

$$Q_w = Q_{w1} + Q_{w2} \tag{24}$$
$$Q_{w1} \le Cap_1 \tag{25}$$
$$Q_{w2} \le Cap_2 \tag{26}$$
$$R_{s\eta} \ge -1, \qquad \eta = 1,2 \tag{27}$$
$$R_w \ge -1 \tag{28}$$
$$R_r \ge 0 \tag{29}$$

In Eq. (23), TC_i represents the system cost per time unit for Case i and P_i represents the probability of Case i occurance ($i = 1,2,3,4$). Eq. (24) ensures that the placed order from warehouse at the two suppliers is exactly divided between the two considered suppliers. Eqs. (25)-(26) ensure that placed orders at the suppliers do not exceed the maximum capacity of the suppliers. Eqs. (27)-(28) denote the reorder point at the warehouse and the suppliers is supposed to be equal or greater than -1. Eq. (29) prohibits the occurrence of order crossover at the retailers as there is no more than one outstanding order at any time.

2.3 Approximating demand rate at the warehouse and suppliers

It is required to determine λ_w, λ_{s1} and λ_{s2} in order to evaluate the cost function. We present an algorithm to obtain the demand rate at the warehouse and suppliers. Due to the stock out at each supplier, warehouse orders are delivered with a random delay. Also, as backorders occur at the warehouse, retailers' orders are replenished with a random delay. Delayed orders result in unsatisfied demands to be lost at the retailers; thus, the demand rate and backordered demands decrease at the warehouse and suppliers. The backordered demands and demand rate at the warehouse and the suppliers have a proportional relation together which can be calculated through the following algorithm (Table. 3):

DOI: 10.24818/18423264/54.4.20.15

Table 3. Presented algorithm to approximate the demand rate at the warehouse and suppliers

Step	Description
Step 1:	$\mathrm{n}=0$, Calculate the initial demand rate at the warehouse from $\lambda_{\mathrm{w}}^{1}=\mathrm{N}\lambda_{\mathrm{r}}$
Step 2:	$\mathrm{n}=\mathrm{n}+1$, Obtain the demand rate at supplier 1 and supplier 2 from $\lambda_{\mathrm{s}1}^{\mathrm{n}}=\lambda_{\mathrm{s}2}^{\mathrm{n}}=\frac{\lambda_{\mathrm{w}}^{\mathrm{n}}}{\mathrm{Q}_{\mathrm{w}}}$
Step 3:	Determine the expected backorder at the supplier 1 $\mathrm{B}_{\mathrm{s}1}^{\mathrm{n}}=\sqrt{(\mathrm{R}_{\mathrm{s}1}-\lambda_{\mathrm{s}1}\mathrm{L}_{\mathrm{s}1})}[\varphi(\mathrm{z}_{\mathrm{s}1})-\mathrm{z}_{\mathrm{s}1}(1-\Phi(\mathrm{z}_{\mathrm{s}1}))$ While, in the above equation, $\varphi(.)$ and $\Phi(.)$ are, respectively, normal density and cumulative distribution functions.
Step 4:	Compute average lead time from supplier 1 to the warehouse by $\bar{\mathrm{L}}_{\mathrm{w}1}^{\mathrm{n}}=\mathrm{L}_{\mathrm{w}1}+\frac{\mathrm{B}_{\mathrm{s}1}^{\mathrm{n}}}{\lambda_{\mathrm{s}1}^{\mathrm{n}}}$
Step 5:	Determine the expected backorder at the supplier 2 $\mathrm{B}_{\mathrm{s}2}^{\mathrm{n}}=\sqrt{(\mathrm{R}_{\mathrm{s}2}-\lambda_{\mathrm{s}2}\mathrm{L}_{\mathrm{s}2})}[\varphi(\mathrm{z}_{\mathrm{s}2})-\mathrm{z}_{\mathrm{s}2}(1-\Phi(\mathrm{z}_{\mathrm{s}2}))]$
Step 6:	Compute average lead time from supplier 2 to the warehouse by $\bar{\mathrm{L}}_{\mathrm{w}2}^{\mathrm{n}}=\mathrm{L}_{\mathrm{w}2}+\frac{\mathrm{B}_{\mathrm{s}2}^{\mathrm{n}}}{\lambda_{\mathrm{s}2}^{\mathrm{n}}}$
Step 7:	Calculate the effective lead time at the warehouse $\mathrm{L}_{\mathrm{w}}^{\mathrm{n}}=\min(\bar{\mathrm{L}}_{\mathrm{w}1}^{\mathrm{n}},\bar{\mathrm{L}}_{\mathrm{w}2}^{\mathrm{n}})$
Step 8:	Obtain the expected backorder at the warehouse $\mathrm{B}_{\mathrm{w}}^{\mathrm{n}}=\sqrt{(\mathrm{R}_{\mathrm{w}}-\lambda_{\mathrm{w}}\mathrm{L}_{\mathrm{w}})}[\varphi(\mathrm{z}_{\mathrm{w}})-\mathrm{z}_{\mathrm{w}}(1-\Phi(\mathrm{z}_{\mathrm{w}}))]$
Step 9:	Compute average lead time from the warehouse to the retailer by $\bar{\mathrm{L}}_{\mathrm{r}}^{\mathrm{n}}=\mathrm{L}_{\mathrm{r}}+\frac{\mathrm{B}_{\mathrm{w}}^{\mathrm{n}}}{\lambda_{\mathrm{w}}^{\mathrm{n}}}$
Step 10:	Compute the expected length of time per cycle that the retailer is out of stock by $\mathrm{T}_{\mathrm{r}}^{\mathrm{n}}=\mathrm{L}_{\mathrm{r}}\times\mathrm{P}(\mathrm{R}_{\mathrm{r}},\lambda_{\mathrm{r}}\bar{\mathrm{L}}_{\mathrm{r}}^{\mathrm{n}})-\frac{\mathrm{R}_{\mathrm{r}}}{\lambda_{\mathrm{r}}}\times\mathrm{P}(\mathrm{R}_{\mathrm{r}}+1,\lambda_{\mathrm{r}}\bar{\mathrm{L}}_{\mathrm{r}}^{\mathrm{n}})$
Step 11:	Update the demand rate at the warehouse by $\lambda_{\mathrm{w}}^{\mathrm{n}+1}=\frac{\mathrm{N}\lambda_{\mathrm{r}}}{\mathrm{Q}_{\mathrm{r}}+\lambda_{\mathrm{r}}\mathrm{T}_{\mathrm{r}}^{\mathrm{n}}}$
Step 12:	Convergence evaluation $\lvert\lambda_{\mathrm{w}}^{\mathrm{n}+1}-\lambda_{\mathrm{w}}^{\mathrm{n}}\rvert<\varepsilon$, otherwise, return to Step 2

3. Numerical examples

A number of numerical examples have been given as in Table 4 in order to evaluate the accuracy of the proposed model. A three-echelon supply chain including two suppliers, a central warehouse and five identical and independent retailers is considered. The retailers face Poisson demand. A basic numerical example is designed; the values of parameters are considered as: $N=5$, $L_{w1}=1$, $L_{w2}=1.5$, $Cap_1=6$ and $Cap_2=5$. In order to determine the effect of the parameters on the results, the basic example is modified by varying some parameters as $\lambda_r=3{,}5{,}7$, $h_r=1{,}2$, $\beta_r=5{,}25$, $h_w=1{,}2$, $h_{si}=1{,}2$, $L_{s1}=1.5{,}3$ and $L_{s2}=2{,}4$.

DOI: 10.24818/18423264/54.4.20.15

Table 4. Numerical problems related to Retailers, Warehouses and Suppliers

No	λ_r	β_w	β_r	h_r h_w h_{si}	L_{s1}, L_{s2}	No	λ_r	β_w	β_r	h_r h_w h_{si}	L_{s1}, L_{s2}
1	3	5	5	1	(1.5,2)	25	3	5	5	1	(3,4)
2	3	25	5	1	(1.5,2)	26	3	25	5	1	(3,4)
3	3	5	2	1	(1.5,2)	27	3	5	25	1	(3,4)
4	3	25	2	1	(1.5,2)	28	3	25	25	1	(3,4)
5	3	5	5	2	(1.5,2)	29	3	5	5	2	(3,4)
6	3	25	5	2	(1.5,2)	30	3	25	5	2	(3,4)
7	3	5	2	2	(1.5,2)	31	3	5	25	2	(3,4)
8	3	25	2	2	(1.5,2)	33	3	25	25	2	(3,4)
9	5	5	5	1	(1.5,2)	33	5	5	5	1	(3,4)
10	5	25	5	1	(1.5,2)	34	5	25	5	1	(3,4)
11	5	5	2	1	(1.5,2)	35	5	5	25	1	(3,4)
12	5	25	2	1	(1.5,2)	36	5	25	25	1	(3,4)
13	5	5	5	1	(1.5,2)	37	5	5	5	1	(3,4)
14	5	25	5	1	(1.5,2)	38	5	25	5	1	(3,4)
15	5	5	2	1	(1.5,2)	39	5	5	25	1	(3,4)
16	5	25	2	1	(1.5,2)	40	5	25	25	1	(3,4)
17	7	5	5	2	(1.5,2)	41	7	5	5	2	(3,4)
18	7	25	5	2	(1.5,2)	42	7	25	5	2	(3,4)
19	7	5	2	2	(1.5,2)	43	7	5	25	2	(3,4)
20	7	25	2	2	(1.5,2)	44	7	25	25	2	(3,4)
21	7	5	5	1	(1.5,2)	45	7	5	5	1	(3,4)
22	7	25	5	1	(1.5,2)	46	7	25	5	1	(3,4)
23	7	5	2	1	(1.5,2)	47	7	5	25	1	(3,4)
24	7	25	2	1	(1.5,2)	48	7	25	25	1	(3,4)

Optimal ordering policies including the order quantity and reorder point at the retailers, warehouse and suppliers have been obtained using Optimization toolbox; then, we have approximated the total cost of the given inventory system including holding costs at all echelons (retailers, central warehouse and suppliers), backorder cost at the warehouse and lost sale cost at the retailers. As simulation of the systems makes it possible to experiment the real situation with the purpose of getting the information about the proposed system; therefore, the accuracy of the results is then evaluated by simulation (Seifbarghy and Esfandiari, 2010). The simulation time length is considered to be 110000 unit times with 10000 unit times

DOI: 10.24818/18423264/54.4.20.15

as a warm-up period. The performance of the model is evaluated by comparing the cost of the mathematical model with the simulation cost.

Table 5. Total cost of inventory system for numerical problems

No	$(Q_{s1},R_{s1},Q_{s2},R_{s2},Q_w,R_w,Q_{w1},Q_{w2},Q_r,R_r)$	Math Model	Simulation	Error
1	(3,1,2,2,3,1,2,1,2,2)	66.74	69.60	0.04
2	(2,-1,1,1,2,-1,1,1,1,2)	164.88	152.88	0.08
3	(3,2,2,1,2,2,1,1,3,2)	155.08	155.41	0.00
4	(3,2,3,2,3,1,2,1,1,1)	289.38	324.49	0.11
5	(2,1,1,1,4,1,3,1,2,2)	81.45	83.42	0.02
6	(3,1,1,1,2,3,1,1,1,1)	110.10	116.82	0.06
7	(3,-1,3,2,3,1,2,1,2,2)	235.48	221.25	0.06
8	(2,2,3,2,3,2,1,2,1,1)	291.70	322.43	0.10
9	(2,2,2,1,3,-1,1,2,1,1)	128.77	129.03	0.00
10	(3,2,2,1,4,1,1,3,2,1)	149.51	159.20	0.06
11	(3,2,2,1,4,1,1,3,2,1)	419.03	463.87	0.10
12	(3,-1,2,1,4,2,2,2,2,2)	593.37	653.03	0.09
13	(3,1,1,1,3,-1,2,1,1,1)	114.50	119.24	0.04
14	(3,1,1,1,3,-1,2,1,1,1)	188.54	193.12	0.02
15	(3,2,4,3,3,1,2,1,2,1)	481.49	464.30	0.04
16	(2,-1,2,1,4,1,2,2,3,1)	527.16	566.11	0.07
17	(2,2,3,1,3,0,2,1,1,0)	177.96	173.04	0.03
18	(3,1,3,2,5,1,3,2,1,1)	179.44	207.51	0.14
19	(3,-1,2,2,3,1,2,1,2,2)	673.57	678.93	0.01
20	(2,1,2,2,4,-1,3,1,2,1)	864.02	825.23	0.05
21	(3,1,3,1,2,1,1,1,1,1)	170.10	172.79	0.02
22	(3,2,2,1,3,-1,2,1,2,1)	340.52	302.35	0.13
23	(2,1,2,1,3,-1,1,2,2,1)	805.16	769.04	0.05
24	(3,1,3,2,5,1,3,2,1,1)	787.71	818.06	0.04
25	(3,-1,1,1,2,-1,1,1,3,2)	121,85	129.91	0.06
26	(3,2,1,-1,3,2,2,1,1,0)	143.11	132.43	0.08
27	(4,1,3,2,2,1,1,1,2,2)	271.83	280.89	0.03
28	(4,2,2,1,4,2,1,3,1,0)	353.51	351.86	0.00
29	(3,1,2,2,2,2,1,1,3,1)	83.17	79.85	0.04
30	(2,2,2,1,4,2,1,3,1,2)	130.59	120.19	0.09
31	(2,1,3,2,2,-1,1,1,1,2)	337.67	341.34	0.01
33	(3,2,5,2,3,3,2,1,1,1)	315.11	323.70	0.03
33	(2,2,1,-1,3,2,1,2,1,2)	129.10	134.94	0.04
34	(3,1,2,-1,2,1,1,1,2,1)	261.08	290.05	0.10
35	(5,3,4,2,2,1,1,1,2,2)	452.51	497.90	0.09
36	(3,1,2,1,3,2,1,2,1,1)	567.44	625.57	0.09
37	(3,2,3,1,4,3,1,3,1,2)	113.84	122.65	0.07

DOI: 10.24818/18423264/54.4.20.15

38	(5,1,1,-1,5,1,3,2,1,1)	190.65	203.97	0.07
39	(2,-1,2,1,4,1,1,3,3,2)	482.90	481.05	0.00
40	(2,1,2,2,4,1,1,3,2,1)	567.48	599.06	0.05
41	(5,2,1,1,2,1,1,1,2,1)	179.77	192.72	0.07
42	(2,1,4,-1,4,1,1,3,1,1)	223.03	251.27	0.11
43	(2,2,4,2,2,1,1,1,3,2)	690.25	705.54	0.02
44	(3,2,3,2,4,3,2,2,1,2)	785.95	810.51	0.03
45	(5,2,1,1,2,-1,1,1,2,2)	247.32	231.75	0.07
46	(3,1,2,-1,3,1,1,2,1,1)	260.02	260.71	0.00
47	(3,2,4,2,3,1,1,2,2,1)	710.23	740.44	0.04
48	(5,1,2,2,3,1,1,2,2,1)	827.67	882.35	0.06
				5.35%

As illustrated in Table (5), the average error of the mathematical model compared to the simulation is relatively low and equal to 5.4%. The total cost of the mathematical model has no significant difference (with P-value = 0.455) with the simulation cost and is of a high accuracy. Supposing different values for retailer's demand rate ($\lambda_r = 3,5,7$) the average error is 5.09%, 5.8% and 5.3%, respectively. However, when $L_{s1} = 1.5$ and $L_{s2} = 2$, the average error is 5.5% and when $L_{s1} = 3$ and $L_{s2} = 4$ the average error decreases to 5.2%.

4. Conclusions and future research

In this paper, the total cost function of a three-echelon inventory system with continuous review policy has been approximated. The given supply chain consists of two suppliers, a central warehouse and a number of retailers; retailers face independent and identical Poisson demand while unsatisfied demand is lost at the retailers. Transportation times between all facilities are constant while random delay may occur due to the stock out at the suppliers and warehouse. Due to the stochastic nature of the delay, lead time at the warehouse and retailers is of randomness; thus, adding two suppliers as third echelon as well as stochastic lead time causes order crossover at the warehouse. It is not straightforward to derive the cost function; the initial problem should be divided into a number of sub-problems. Based on the delay value at each supplier, four different cases were considered to handle the order crossover. We derived an approximate cost function for each case based on the mean of costs for one-for-one policies. Finally, the total cost function of the initial problem was derived based on the weighted average cost of the given cases.

Using the mean of costs for one-for-one policies is more straightforward than applying the demand distribution during the lead time. Numerical examples with relatively low errors confirmed the accuracy of the presented model. In this paper, the optimal ordering policies are local optimum and the order quantity at the higher level is assumed to be multiple of the order quantity at the lower level. As

DOI: 10.24818/18423264/54.4.20.15

future research, we can consider a decentralized system and using different types of game theory approaches in order to find order quantity and reorder points.

Appendix

In this section, we will give short summery of the retailer's cost per unit while the first arriving order at the warehouse comes from supplier *1*. A three-echelon supply chain including one supplier, a central warehouse and single retailer is considered while their ordering policies are (R_{s1}, Q_{s1}), (R_w, Q_{w1}) and (R_r, Q_r), respectively. The inventory positions at the retailer, warehouse and supplier *1* are i, l and $kQ_{w1}Q_r$, respectively. Per each item in a batch, the inventory position at supplier *1* is $kQ_{w1}Q_r$ and k has been distributed uniformly between the two values of $R_{s1}+1$ and $R_{s1}+Q_{s1}$. Besides, the inventory position at the retailer varies between the values of R_r+1 and R_r+Q_r.

In this supply chain, there is more than one retailer and both the retailers and the warehouse (as the higher level) order in batches, each customer demand may trigger a retailer order from the warehouse. The l_{th} customer demand (after the warehouse order) will trigger the j_{th} subsequent retailer order by p_{lj} while j has been distributed uniformly between R_w+1 and R_w+Q_w.

Therefore, the retailer's cost per unit for units replenished from supplier *1* can be given as in A.1:

$$K1=\frac{1}{Q_{s1}Q_{w1}Q_r}\sum_{k=R_{s1}+1}^{R_{s1}+Q_{s1}}\sum_{i=R_r+1}^{R_r+Q_r}\left(\sum_{j=(0,R_w+1)^+}^{R_w+Q_{w1}}c_{ijk}^1+\sum_{j=1}^{-R_w-1}d_{ijk}^1\right) \qquad \text{A.1}$$

Similarly, the retailer's cost per unit for items replenished from supplier *2* is given by A.2:

$$K2=\frac{1}{Q_{s2}Q_{w2}Q_r}\sum_{k=R_{s2}+1}^{R_{s2}+Q_{s2}}\sum_{i=R_r+1}^{R_r+Q_r}\left(\sum_{j=(0,R_w+Q_{w1}+1)^+}^{R_w+Q_w}c_{ijk}^2+\sum_{j=1}^{-R_w-1}d_{ijk}^2\right) \qquad \text{A.2}$$

As the share of supplier *1* in order replenishment at the warehouse is Q_{w1}/Q_w and the share of supplier *2* is Q_{w2}/Q_w, the retailer's cost per unit is obtained from A.3:

DOI: 10.24818/18423264/54.4.20.15

$$K_1^r = \frac{Q_{w1}}{Q_w} \times \frac{1}{Q_{s1}Q_{w1}Q_r} \sum_{k=R_{s1}+1}^{R_{s1}+Q_{s1}} \sum_{i=R_r+1}^{R_r+Q_r} \left(\sum_{j=(0,R_w+1)^+}^{R_w+Q_{w1}} c_{ijk}^1 + \sum_{j=1}^{-R_w-1} d_{ijk}^1 \right) + \frac{Q_{w2}}{Q_w} \times \frac{1}{Q_{s2}Q_{w2}Q_r} \sum_{k=R_{s2}+1}^{R_{s2}+Q_{s2}} \sum_{i=R_r+1}^{R_r+Q_r} \left(\sum_{j=(0,R_w+Q_{w1}+1)^+}^{R_w+Q_w} c_{ijk}^2 + \sum_{j=1}^{-R_w-1} d_{ijk}^2 \right) \quad \text{A.3}$$

and finally, the retailer's cost per unit is obtained from A.4:

$$K_1^r = \frac{1}{Q_{s1}Q_wQ_r} \sum_{k=R_{s1}+1}^{R_{s1}+Q_{s1}} \sum_{i=R_r+1}^{R_r+Q_r} \left(\sum_{j=(0,R_w+1)^+}^{R_w+Q_{w1}} c_{ijk}^1 + \sum_{j=1}^{-R_w-1} d_{ijk}^1 \right) + \frac{1}{Q_{s2}Q_wQ_r} \sum_{k=R_{s2}+1}^{R_{s2}+Q_{s2}} \sum_{i=R_r+1}^{R_r+Q_r} \left(\sum_{j=(0,R_w+Q_{w1}+1)^+}^{R_w+Q_w} c_{ijk}^2 + \sum_{j=1}^{-R_w-1} d_{ijk}^2 \right) \quad \text{A.4}$$

REFERENCES

[1] **Abginehchi, S., Farahani, R. Z., Rezapour, S. (2013),** ***A Mathematical Model for Order Splitting in a Multiple Supplier Single-Item Inventory System****; Journal of Manufacturing Systems,* 32(1): 55–67;
[2] **Axsater, S. (1990),** ***Simple Solution Procedures for a Class of Two-echelon Inventory Problem****; European Journal of Operations Research,* 38(1):64-69;
[3] **Axsater S. (1993),** ***Exact and Approximate Evaluation of Batch-ordering Policies for Two-level Inventory Systems****. European Journal of Operational Research,* 41(4): 777-785;
[4] **Bagul, A. and Mukherjee, I. (2019),** ***Centralized vs. Decentralized Sourcing Strategy for Multi-tier Automotive Supply Network****; International Journal of Productivity and Performance Management,* 68(3): 578-607;
[5] **Cao, P., Yao, D. (2019),** ***Dual Sourcing Policy for a Continuous-Review Stochastic Inventory System***; *IEEE Transactions on Automatic Control*, 64(7): 2921-2928;

DOI: 10.24818/18423264/54.4.20.15

[6] **Chang, S.C., Chang, C.T. (2017),** ***A Multi-stage and Multi-supplier Inventory Model Allowing Different Order Quantities*****;** *Applied Mathematical Modelling*, 52: 613-625;
[7] **Duan, L., Ventura, J. A. (2019),** ***A Dynamic Supplier Selection and Inventory Management Model for a Serial Supply Chain with a Novel Supplier Price Break Scheme and Flexible Time Periods***; *European Journal of Operational Research*, 272(3): 979-998;
[8] **Fong, D.K.H., Gempeshaw, V.M., Ord, K.J. (2000),** ***Analysis of a Dual Sourcing Inventory Model with Normal Unit Demand and Erlang Mixture Lead Times****; European Journal of Operational Research*. 120(1): 97–107;
[8] **Hajiaghaei-Keshteli, M., Sajadifar, S. M. (2010),** ***Deriving the Cost Function for a Class of Three-echelon Inventory System with N-Retailers and One-for-One Ordering Policy***; *International Journal of Advanced Manufacturing Technology*, 50: 343–351;
[10] **Hill, M.R. (1996),** ***Order Splitting in Continuous Review (Q, R) Inventory Models****; European Journal of Operational Research*, 95(1): 53-61;
[11] **Kok, T.D., Grob, C., Laumanns, M., Minner, S., Rambau, J., Schade, K. (2018),** ***A Typology and Literature Review on Stochastic Multi-echelon Inventory Models****; European Journal of Operational Research*, Vol. 269, No. 1. 955–983;
[12] **Konur, D., Campbell, J.F., Monfared, S.A. (2016),** ***Economic and Environmental Considerations in A Stochastic Inventory Control Model with Order Splitting under Different Delivery Schedules Among Suppliers****; Omega*. 80: 142-165;
[13] **Ma, X, Rossi, R, Archibald, T. (2019),** ***Stochastic Inventory Control: A Literature Review***. In: *Proceedings of the 9th IFAC INFORMS Conference: Manufacturing Modelling, Management and Control - MIM 2019.* Berlin, Germany, 1490-1495;
[14] **Riezebos, J. (2006),** ***Inventory Order Crossovers****; International Journal of Production Economics*, 104(2): 666–675;
[15] **Sculli, D., Wu, S. (1981),** ***Stock Control with Two Suppliers and Normal Lead Times***. *Journal of the Operational Research Society*, 32(11): 1003–1009;
[16] **Seifbarghy, M., Esfandiari, N. (2010),** ***Cost Evaluation of a Two-echelon Inventory System with Lost Sales and Non-Identical Retailers***; *Economic Computation and Economic Cybernetics Studies and Research*, 44(4): 199-212;
[17] **Song, D.P., Dong, J.X., Xu, J. (2014),** ***Integrated Inventory Management and Supplier Base Reduction in a Supply Chain with Multiple Uncertainties****; European Journal of Operational Research*, 232, 522–536.

DOI: 10.24818/18423264/54.4.20.15